Measuring Humidity - Basics

Dalton’s Law

Air is a mixture of different gases. Under normal environmental conditions the gases have an ideal behaviour, i.e. each gas molecule can act independently from all others. **Dalton’s law** is valid:

The total pressure of a gas is the sum of the partial pressures

\[ p \ [\text{mbar, hPa}] = p_{N_2} + p_{O_2} + p_{Ar} + \ldots \]

The partial pressure \( p \) is defined as the pressure of a gas, if it would occupy alone the whole volume of the gas mixture.

Water in its gaseous phase (vapour) is also a component of air mixture. Under normal conditions it behaves like an ideal gas. With Dalton’s law \( p \) becomes:

\[ p \ [\text{mbar, hPa}] = p_{N_2} + p_{O_2} + p_{Ar} + \ldots + e \]

or

\[ p \ [\text{mbar, hPa}] = p_{da} + e \]

\( e \) — partial pressure of (water) vapour

\( p_{da} \) — partial pressure of dry air

Vapour Pressure Above Liquid

The concentration of water vapour in air is limited. There is a maximum partial pressure of vapour which depends on temperature. Air at high temperature can take more vapour than at low temperature.

This behaviour can be explained as follows:

The molecules in a liquid are moving with different velocities (or energies) whereby the average energy is proportional to the temperature of the liquid. With respect to energy, the water molecules show a statistical distribution as in Fig. 1.

The molecules with energy lower than the binding energy of the liquid cannot leave the water surface. Those with higher energy can leave the water. They evaporate and increase the vapour partial pressure in the air (Fig. 2).

The opposite phenomenon happens with the water vapour molecules. Those with lower energy than the binding level of the liquid condensate on the water surface and decrease the vapour partial pressure in the air.

![Fig.1: Statistical energy distribution of molecules in a liquid.](image1)

![Fig.2: Equilibrium at temperature T.](image2)
In a closed volume partly filled with water at temperature $T$ (Fig.2) there is an equilibrium between evaporation and condensation. If there is a lack of water molecules in the moist region, more evaporation will occur and the vapour concentration will increase. In the opposite case more molecules will condense than evaporate and the vapour concentration will decrease.

The balance between evaporation and condensation leads to a vapour partial pressure (respectively concentration) which only depends on temperature.

A temperature rise will increase the energy of water molecules (Fig.1) and the balance will be shifted to higher vapour concentration.

For equilibrium at temperature $T$ the vapour concentration (or water partial pressure $e$ or number of water molecules per $m^3$) is the maximum concentration which can exist at this temperature and cannot be exceeded. A higher concentration would lead to condensation again and after a short time the old balance would be reached.

This vapour concentration is called saturated concentration or in terms of partial pressure saturation vapour pressure above water $e_{ws}$ at temperature $T$.

The saturation pressure above water $e_{ws}$ has an exponential dependence on $T$ and is given in Tab.1.

### Vapour Pressure Above Ice

Below 0.01 degC (triple point of water) water can exist in a liquid phase as well as in a solid phase (ice) whereby the liquid phase is not stable. For temperatures lower than 0.01 degC, in addition to vapour pressure above water there is also a vapour pressure above ice. (Tab 2.)

<table>
<thead>
<tr>
<th>$T$ [degC]</th>
<th>$e_{ws}$ [mbar]</th>
<th>$T$ [degC]</th>
<th>$e_{ws}$ [mbar]</th>
<th>$T$ [degC]</th>
<th>$e_{is}$ [mbar]</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1014.19</td>
<td>0</td>
<td>6.112</td>
<td>0.01</td>
<td>6.117</td>
</tr>
<tr>
<td>90</td>
<td>701.82</td>
<td>-10</td>
<td>2.8652</td>
<td>0</td>
<td>6.112</td>
</tr>
<tr>
<td>80</td>
<td>474.16</td>
<td>-20</td>
<td>1.2559</td>
<td>-10</td>
<td>2.5989</td>
</tr>
<tr>
<td>70</td>
<td>312.02</td>
<td>-30</td>
<td>0.5103</td>
<td>-20</td>
<td>1.0324</td>
</tr>
<tr>
<td>60</td>
<td>199.48</td>
<td>-40</td>
<td>0.1903</td>
<td>-30</td>
<td>0.3800</td>
</tr>
<tr>
<td>50</td>
<td>123.53</td>
<td>-50</td>
<td>0.0644</td>
<td>-40</td>
<td>0.1284</td>
</tr>
<tr>
<td>40</td>
<td>73.853</td>
<td>-60</td>
<td>0.0195</td>
<td>-50</td>
<td>0.0394</td>
</tr>
<tr>
<td>30</td>
<td>42.470</td>
<td>-70</td>
<td>5.187E-03</td>
<td>-60</td>
<td>0.0108</td>
</tr>
<tr>
<td>20</td>
<td>23.392</td>
<td>-80</td>
<td>1.190E-03</td>
<td>-70</td>
<td>2.615E-03</td>
</tr>
<tr>
<td>10</td>
<td>12.281</td>
<td>-90</td>
<td>2.298E-04</td>
<td>-80</td>
<td>5.472E-04</td>
</tr>
<tr>
<td>0.01</td>
<td>6.117</td>
<td>-100</td>
<td>3.622E-05</td>
<td>-90</td>
<td>9.670E-05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-100</td>
<td>1.402E-05</td>
</tr>
</tbody>
</table>

### Tab.1: Saturation vapour pressure values $e_{ws}$ above water.

### Tab.2: Saturation vapour pressure values $e_{is}$ above ice.
Consequently there are two saturation curves below 0.01 degC which are given in Fig.3 in a logarithmic scale.
From -100 degC to 100 degC the saturation vapour pressure is changing over 8 orders of magnitude.

**Real Gas Correction**

Up to now we have regarded water vapour as an ideal gas, i.e. water molecules act independently from each other in the air mixture. In reality there is a small interaction between molecules which leads to a small increase of saturation vapour in air. This is described by an enhancement factor \( f(p,T) \).
For normal pressure the enhancement factor is near one and can be neglected. In this case, water vapour can be seen as an ideal gas.

**Humidity Functions**

**Relative Humidity RH [%RH]**
Tab.1 and Tab.2 give the values for saturation vapour pressure as a function of temperature. These values are maximum values and cannot be exceeded. Usually the partial vapour pressure is lower.

Relative humidity RH is defined as the ratio between the actual partial vapour pressure \( e \) and the saturation vapour pressure above water \( e_{ws} \):

\[
RH = \left( \frac{e}{e_{ws}} \right) \times 100 \quad [%RH]
\]

**Absolute Humidity (vapour density) \( dv \) [g/m³]**
is the mass of water in 1 m³ moist air.

\[
dv = \frac{mv}{v} \quad [g/m³] \quad mv... \text{ mass of water vapour} \quad v... \text{ air volume}
\]

**Dew Point Td [degC]**
When cooling air with non saturated vapour pressure \( e \) the partial pressure will stay constant for a while. Simultaneously the relative humidity will increase due to decreasing saturation vapour pressure:

\[
RH = \left( \frac{e}{e_{ws}} \right) \times 100 \quad e_{ws} = e_{ws}(T) \quad [%] \quad T \text{ decreasing} \implies e_{ws} \text{ decreasing} \implies RH \text{ increasing}
\]

At 100% RH the partial pressure of vapour \( e \) in the air equals the saturation vapour pressure above water \( e_{ws}(T) \):

\[
e = e_{ws}(T)
\]

If the temperature decreases further, condensation will start.

**The dew point Temperature Td is the temperature where condensation begins.**
**Mixing Ratio r [g/kg]**

r is the mass of water to evaporate and mix with 1 kg dry air to perform a certain relative humidity or partial vapour pressure $e$.

**Specific Enthalpy h [kJ/kg]**

The enthalpy of 1 kg moist air with relative humidity $RH$ and corresponding mixing ratio $r$ at temperature $T$ is the total energy you need:

- to warm up dry air from 0 degC to $T$
- to evaporate the water (latent heat of water)
- to warm up the vapour from 0 degC up to $T$

The specific enthalpy is a relative quantity, i.e. only variations are of interest, not the absolute value. The variation of enthalpy is the measure of energy required to transform the moist air from one equilibrium state to another.

**example 1:**
To warm up air from 20 degC to 25 degC and humidify the air from 40% RH to 60% RH 20.2 kJ/kg would be needed.

<table>
<thead>
<tr>
<th>$T$ [degC]</th>
<th>$RH$ [%RH]</th>
<th>$h$ [kJ/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>state 1</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>state 2</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td>difference</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**example 2:**
Warming up from 20 to 25 degC at constant relative humidity 40% requires only 10.3 kJ/kg.

<table>
<thead>
<tr>
<th>$T$ [degC]</th>
<th>$RH$ [%RH]</th>
<th>$h$ [kJ/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>state 1</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>state 2</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>difference</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**example 3:**
Warming up from 20 to 25 degC at constant partial vapour pressure (i.e. $e = \text{const}$, $r = \text{const}$, $Td = \text{const}$), the relative humidity decreases from 40% to 29.5% RH. This requires only 5.1 kJ/kg energy.

<table>
<thead>
<tr>
<th>$T$ [degC]</th>
<th>$RH$ [%RH]</th>
<th>$h$ [kJ/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>state 1</td>
<td>20</td>
<td>40</td>
</tr>
<tr>
<td>state 2</td>
<td>25</td>
<td>29.5</td>
</tr>
<tr>
<td>difference</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The Mollier diagram is a very useful instrument to solve HVAC-problems graphically. It includes all humidity functions in one chart.

**Fig. 4a**: Mollier diagram: curves of constant relative humidity. The region below 100% (fog region) is not valid because condensation occurs.

**Fig. 4b**: Curves of constant enthalpy are added to Fig. 4a. Also example 1 is included.
Accuracies of the calculated variables are subject to the accuracy of the humidity and temperature measurement.

The above accuracy values are valid for ±2% RH and ±0.2 degC.

High accuracy values can be reached by a special humidity/temperature calibration.